working life of the turbines, and here the prediction of the behavior of solid material is particularly important.

NOTATION

Ω_1, domain of spatial variables with boundary Γ; Γ_1 and Γ_3, boundary segments parallel to 0z (inlet and outlet, respectively); Γ_2, free surface; Γ_4, bottom of contour; n, direction of the exterior normal to the boundary of Ω_1; L and H, channel length and depth; x and z, horizontal and vertical coordinates; t', time; U and W, horizontal and vertical velocity components; c', impurity concentration; ω', hydraulic parameter; D_x and D_z, turbulent-diffusion coefficients; F', source (sink) function; $0 \leq \alpha \leq 1$, bottom-absorption coefficient; U_*, dynamic velocity, U_m, free-surface value of U; U_{av}, average value of U; c_1, characteristic inlet value of impurity concentration; $\Delta_1(\Delta_2)$, step size along $0x_1(0x_2)$; δ_{ij}, Kronecker symbol; k, Karman constant.

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CHARACTERISTIC METHOD IN HEAT TRANSPORT IN FAST NONSTATIONARY PROCESSES

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UDC 536.2

The characteristic method has been used in numerical solution of a hyperbolic heat-transport equation.

The following hyperbolic equation is involved in heat-transfer calculation for fast nonstationary processes of one-dimensional type:

$$c\rho \frac{\partial T}{\partial \tau} + c\rho\tau_r \frac{\partial^2 T}{\partial \tau^2} = \lambda \frac{\partial^2 T}{\partial x^2} \quad (1)$$

subject to the appropriate initial and boundary conditions. As a rule, the boundary conditions are nonlinear, and then there are major difficulties in obtaining an analytic solution. A network method (explicit difference scheme) has been used [1] to solve (1). Studies have been made [2,3] on the construction of difference schemes for equations of hyperbolic type on the basis of characteristic relationships, particularly with regard to the stability; here we show that the characteristic method can be applied in heat-transfer calculations for fast nonstationary processes.

We first put

$$V = \frac{\partial T}{\partial \tau}, \quad W = \frac{\partial T}{\partial x}, \quad (1)$$

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to reduce (1) to a system of first-order equations in new variables:

$$\frac{\partial V}{\partial \tau}-\frac{a}{\tau_r}\frac{\partial W}{\partial x}=-\frac{V}{\tau_r};\quad \frac{\partial W}{\partial \tau}-\frac{\partial V}{\partial x}=0. \tag{2}$$

We rewrite (2) in vector form:

$$\frac{\partial u}{\partial \tau}+Au_x=f, \tag{3}$$

where u is a vector function; A, a coefficient matrix; and f, a column vector.

We now determine the eigenvalues of A:

$$\det(A-\mu I)=0. \tag{4}$$

Here μ are the eigenvalues and I is a unit matrix; then

$$\begin{Vmatrix}-\mu & -\frac{a}{\tau_r}\\ -1 & -\mu\end{Vmatrix}=0;\quad \mu_{1,2}=\pm\sqrt{\frac{a}{\tau_r}}. \tag{5}$$

This means that the conditions governing hyperbolic systems are met: the roots μ are real and different [4].

Equation (3) can be transformed to the following characteristic form:

$$\omega_k\left(\frac{du}{d\tau}\right)_k=\omega_k f, \tag{6}$$

where ω_k are the eigenvectors of A.

The following are the components of the eigenvectors:

$$\omega_1=\left\{1,\ \sqrt{\frac{a}{\tau_r}}\right\},\quad \omega_2=\left\{1,\ -\sqrt{\frac{a}{\tau_r}}\right\}.$$

We write the initial system of (6) in terms of Riemann invariants:

$$\frac{dP_k}{d\tau}=\omega_k f,\quad k=1,\ 2, \tag{7}$$

where $P_k = \omega_k u$ are those invariants and

$$\frac{dP_1}{d\tau}=\omega_1 f=-\frac{V}{\tau_r};\quad \frac{dP_2}{d\tau}=\omega_2 f=-\frac{V}{\tau_r}. \tag{8}$$

We introduce the variable $P=\begin{Vmatrix}P_1\\P_2\end{Vmatrix}$ and put the system in the form

$$P=\Omega u, \tag{9}$$

where

$$\Omega=\begin{Vmatrix}\omega_1\\ \omega_2\end{Vmatrix}=\begin{Vmatrix}1 & \sqrt{\frac{a}{\tau_r}}\\ 1 & -\sqrt{\frac{a}{\tau_r}}\end{Vmatrix}.$$

The eigenvectors have been selected to be linearly independent, so Ω has an inverse Ω^{-1}; then from (9) we get the solution as

$$u=\Omega^{-1}P. \tag{10}$$

We use Ω^{-1} to get in terms of Riemann invariants that

$$\begin{Vmatrix}V\\ W\end{Vmatrix}=\begin{Vmatrix}\frac{1}{2} & (P_1+P_2)\\ \frac{1}{2}\sqrt{\frac{\tau_r}{a}} & (P_1-P_2)\end{Vmatrix}.$$

Then (8) is transformed with $P_1 = P$ and $P_2 = R$ as follows:

$$\left(\frac{dP_1}{d\tau}\right)_1 = -\frac{P+R}{2\tau_r}; \quad \left(\frac{dP_2}{d\tau}\right)_2 = -\frac{P+R}{2\tau_r}. \tag{11}$$

A Krank–Nicholson scheme [5] is used to find the final solution in difference form.

We introduce the following difference net in region D{0 < x < L, τ ≥ 0}, in which the temperature function T(x, τ) is defined:

$$x_i = ih, \quad i = 0, 1, 2, \ldots, n, \quad n = \frac{L}{h}, \quad h = \text{const} > 0;$$

$$\tau_p = p\Delta h, \quad p = 1, 2, \ldots, \Delta\tau = \text{const} > 0.$$

We replace the derivatives in (11) by the difference relations giving second-order error, and we then get the following for P_i^{p+1} and R_i^{p+1}:

$$P_i^{p+1} - P_{i-1}^p = -\nu\,[(P+R)|_i^{p+1} + (P+R)_{i-1}^p];$$
$$R_i^{p+1} - R_{i+1}^p = -\nu\,[(P+R)_i^{p+1} + (P+R)|_{i+1}^p]. \tag{12}$$

Then

$$P_i^{p+1} = \frac{1}{1+\nu^{-1}}\,[P_{i-1}^p(\nu^{-1}-\nu) - R_{i-1}^p(1+\nu) - R_{i+1}^p(1-\nu) + \nu P_{i+1}^p]; \tag{13}$$

$$R_i^{p+1} = (\nu^{-1}-1)\,P_{i-1}^p - R_{i-1}^p - (1+\nu^{-1})\,P_i^{p+1}; \quad \nu = \frac{\Delta\tau}{4\tau_r}. \tag{14}$$

The Riemann invariants are

$$P = \left.\frac{dT}{d\tau}\right|_1, \quad R = \left.\frac{dT}{d\tau}\right|_2,$$

which gives us an expression for the temperatures of the internal points:

$$T_i^{p+1} = T_i^p + \frac{\Delta\tau}{2}\,(P_i^{p+1} + P_{i-1}^p). \tag{15}$$

We take the boundary conditions as heat transfer in accordance with Newton's law; we assume that the heat-transfer coefficient is α = f(τ), while the temperature of the environment is $T_{en}(\tau) = F(\tau)$, i.e.,

$$\lambda\,\frac{\partial T(0,\tau)}{\partial x} + \alpha(\tau)\,[T_{en}(\tau) - T(0,\tau)] = 0. \tag{16}$$

We transfer from partial derivatives to total derivatives and use (11) to get for k = 2 that

$$\left.\frac{dR}{d\tau}\right|_2 = -\frac{P+R}{2\tau_r}; \quad \left.\frac{dT}{d\tau}\right|_2 = R;$$
$$\frac{dP}{d\tau} - \frac{dR}{d\tau} = \frac{2b}{\lambda}\left\{[T(0,\tau) - T_{en}(\tau)]\,\alpha(\tau) - \left[\dot{T}_{en}(\tau) - \frac{P+R}{2}\right]\alpha(\tau)\right\}. \tag{17}$$

We replace the derivatives in (17) by difference relations and solve the system to get

$$T_1^{p+1} = \frac{2\left[(1+\nu^{-1})\,R_2^p + \frac{2}{\Delta\tau}\left(1+\frac{\nu^{-1}}{2}\right)T_2^p - P_2^p\right] - C}{\frac{4}{\Delta\tau}\left(1+\frac{\nu^{-1}}{2} - \frac{\alpha(\tau)\,b}{\lambda}\,\tau_r\right) + \frac{\alpha(\tau)\,b}{\lambda}\,\Delta\tau};$$

$$P_1^{p+1} = 2\nu^{-1}R_2^p + \frac{2}{\Delta\tau}\,(1+\nu^{-1})\,(T_2^p - T_1^{p+1}) - P_2^p; \tag{18}$$

$$R_1^{p+1} = \frac{2}{\Delta\tau}\,(T_1^{p+1} - T_2^p) - P_2^p,$$

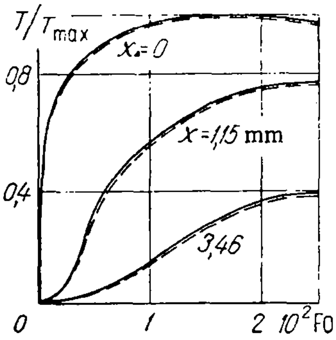

Fig. 1. Temperature variation at several points on a plate of thickness 20 mm for boundary conditions of the third kind (the solid lines are from numerical solution of (1), while the broken lines are from the solution to the heat-conduction equation).

where

$$C=\frac{b\Delta\tau}{\lambda}\left\{\frac{\alpha(\tau)}{2}\left[\frac{2\nu^{-1}}{\Delta\tau}T_2^p-(1-2\nu^{-1})R_2^p-P_2^p\right]-\dot{\alpha}(\tau)T_{\text{en}}(\tau)|_1^{p+1}-\right.$$

$$\left.-\alpha(\tau)\dot{T}_{\text{en}}(\tau)|_1^{p+1}+\left[\dot{\alpha}(\tau)(T_{\text{sur}}-T_{\text{en}}(\tau))|_2^p-\alpha(\tau)\left(\dot{T}_{\text{en}}(\tau)-\frac{P+R}{2}\right)\Big|_2^p\right]\right\}.$$

Here $b = \sqrt{a/\tau_r}$.

Equations (18) allow one to calculate the temperatures at the boundary points at which the heat is input; the temperature at the unheated surface is taken as constant. Transformations analogous to those performed above must be used in other cases.

If α = const, the values of P and R are defined by the following expressions, along with the temperature at the boundary:

$$P_1^{p+1}=\frac{8h\tau_r(1+\nu)B}{(\lambda+\alpha h)[4\tau_r(1+\nu)-\Delta\tau]};$$

$$R_1^{p+1}=\frac{1}{1+\nu}(R_2^p-\nu P_2^p-\nu R_2^p)-\frac{2h\Delta\tau B}{(\lambda+\alpha h)[4\tau_r(1+\nu)-\Delta\tau]};$$

$$T_1^{p+1}=T_2^p+\frac{\Delta\tau}{2}(R_1^{p+1}+R_2^p),$$

where

$$B=\frac{\lambda}{2h}(P_2^{p+1}+R_2^{p+1})+\alpha\dot{T}_{\text{en}}(\tau)-\frac{\lambda-\alpha h}{2h(1+\nu)}(R_2^p-\nu P_2^p-\nu R_2^p).$$

Figure 1 gives computer results from solving (1) subject to the boundary condition of (16) and for initial condition $T(x, 0) = T_0$ (solid lines) for the following intput data: λ = 17 W/m·deg K; c = 0.63 kJ/kg·deg K; ρ = 7.8 g/cm³; $\tau_r = 2\cdot 10^{-10}$ sec; τ = 0-3 sec, with $\alpha(\tau)$ varying linearly from 5.76 to 8.06 kW/m²·deg K and $T_{en}(\tau)$ varying linearly from 3000 to 2000°K.

Figure 1 also shows for comparison results from solving the heat-conduction equation. with the above parameters and conditions (broken lines). Clearly, the two solutions differ only slightly. The results of [3] and the present ones together show that the method and the difference scheme are stable.

NOTATION

λ, thermal conductivity; c, specific heat; ρ, density; T, temperature; τ_r, thermal-stress relaxation time; $\Delta\tau$, time step; α, heat-transfer coefficient.

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COUPLED DYNAMIC THERMOELASTICITY PROBLEM FOR A HALF SPACE WITH THERMAL "MEMORY"

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UDC 539.3

The coupled dynamic thermoelasticity problem is solved for a half space endowed with thermal "memory." The properties of the generated thermoelastic waves are discussed.

Chen and Gurtin [1], elaborating the general nonlinear theory of conduction of Gurtin and Pipkin [2], extended it to include strain in the medium. They derived nonlinear functional defining relations for the thermoviscoelasticity of bodies with thermal and strain memory, whereby the prior history of the variation of the thermodynamic and mechanical characteristics is taken into consideration:

$$\psi(X,\ t)=\Psi(\Lambda^t),\ \ \sigma(X,\ t)=\Sigma(\Lambda^t),\ \ \eta(X,\ t)=N(\Lambda^t),\ q(X,\ t)=Q(\Lambda^t), \tag{1}$$

where $\Lambda^t = (F,\ T,\ \overline{F}^t,\ \overline{T}^t,\ \overline{g}^t)$ is the thermal history of the process (1).

In the present article, we investigate the one-dimensional coupled dynamic problem for a linear thermoelastic isotropic half space. After linearization of the system of defining relations (1) with regard for the laws of conservation of momentum and energy, we arrive at the following dimensionless system of equations for the temperature, stress, and displacement fields induced in the half space:

$$M^2\frac{\partial^2\theta}{\partial\tau^2}+\frac{\partial\theta}{\partial\tau}+\int\limits_0^\infty\beta'(s)\,\frac{\partial\theta(x,\ \tau-s)}{\partial\tau}\,ds=\frac{\partial^2\theta}{\partial x^2}+\int\limits_0^\infty\alpha'(s)\,\frac{\partial^2\theta(x,\ \tau-s)}{\partial x^2}-\varepsilon\,\frac{\partial^3 u}{\partial x\partial\tau^2},$$

$$\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2 u}{\partial\tau^2}=\frac{\partial\theta}{\partial x}-\Gamma\int\limits_0^\infty\gamma(s)\,\frac{\partial\theta(x,\ \tau-s)}{\partial x}\,ds,$$

$$\sigma_x=\frac{\partial u}{\partial x}-\theta+\Gamma\int\limits_0^\infty\gamma(s)\,\theta(x,\ \tau-s)\,ds, \tag{2}$$

$$\sigma_y=\sigma_z=\frac{\varkappa_4}{2\varkappa_3+\varkappa_4}\,\sigma_x-\frac{2\varkappa_3}{2\varkappa_3+\varkappa_4}\left[\theta-\Gamma\int\limits_0^\infty\gamma(s)\,\theta(x,\ \tau-s)\,ds\right],$$

where

$$\varkappa_1\delta_{ij}=D_F\,E\,(\Lambda_0),\ \ -\varkappa_2\delta_{ij}=D_\theta\Sigma(\Lambda_0),\ \ \varkappa_3(\delta_{ij}\delta_{kl}+\delta_{il}\delta_{jk})+\varkappa_4\delta_{ij}\delta_{kl}=D_F\,\Sigma(\Lambda_0).$$

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